
CANDIDATE NUMBER

SYDNEY GRAMMAR SCHOOL

2014 Trial Examination

FORM VI

MATHEMATICS 2 UNIT

Friday 1st August 2014

General Instructions

- Reading time — 5 minutes
- Writing time — 3 hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.
- A list of standard integrals is provided at the end of the examination paper.

Total — 100 Marks

- All questions may be attempted.

Section I – 10 Marks

- Questions 1–10 are of equal value.
- Record your solutions to the multiple choice on the sheet provided.

Section II – 90 Marks

- Questions 11–16 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your candidate number on each booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Place your multiple choice answer sheet inside the answer booklet for Question Eleven.
- Write your candidate number on this question paper and submit it with your answers.

Checklist

- SGS booklets — 6 per boy
- Multiple choice answer sheet
- Candidature — 91 boys

Examiner

MLS

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

What is the gradient of the line $6x + 3y - 2 = 0$?

(A) 2

(B) -2

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

QUESTION TWO

What is 5·29784 correct to three significant figures?

(A) 5·29

(B) 5·297

(C) 5·30

(D) 5·298

QUESTION THREE

Which of the following is equal to $\dfrac{1}{\sqrt{5}+2\sqrt{3}}$?

(A) $\dfrac{\sqrt{5}-2\sqrt{3}}{7}$

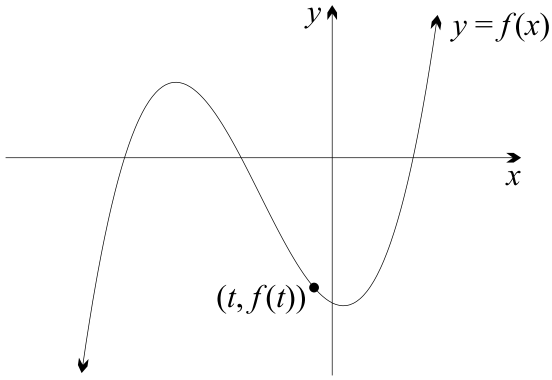
(B) $\dfrac{2\sqrt{3}+\sqrt{5}}{7}$

(C) $\dfrac{2\sqrt{3}-\sqrt{5}}{7}$

(D) $\dfrac{\sqrt{5}+2\sqrt{3}}{-7}$

Exam continues next page ...

QUESTION FOUR



The diagram shows the graph of $y = f(x)$. Which of the following statements is true?

(A) $f'(t) > 0$ and $f''(t) < 0$

(B) $f'(t) > 0$ and $f''(t) > 0$

(C) $f'(t) < 0$ and $f''(t) < 0$

(D) $f'(t) < 0$ and $f''(t) > 0$

QUESTION FIVE

The acceleration of a particle is given by $\ddot{x} = 4\cos 2t$ where x is the displacement in metres and t is time in seconds. Which of the following is a possible expression for its displacement?

(A) $-2\sin 2t$

(B) $2\sin 2t$

(C) $\cos 2t$

(D) $-\cos 2t$

QUESTION SIX

Which of the following is the derivative of $y = \dfrac{e^{7x}}{e^{3x}}$?

(A) $4e^{4x}$

(B) e^{4x}

(C) $\dfrac{7e^{3x}e^{7x} + 3e^{3x}e^{7x}}{e^{9x}}$

(D) $\dfrac{3e^{3x}e^{7x} - 7e^{7x}e^{3x}}{e^{9x}}$

QUESTION SEVEN

A particle moves so that its displacement in metres from the origin at time t seconds is given by $x = 20t - 5t^2$. At what time is it stationary?

(A) 0 seconds

(B) 2 seconds

(C) 4 seconds

(D) 6 seconds

QUESTION EIGHT

How many terms are in the series $31 + 44 + 57 + \cdots + 226$?

(A) 4

(B) 13

(C) 15

(D) 16

QUESTION NINE

Given that $\displaystyle\int_0^4 (x + k)\, dx = 12$ and k is a constant, what is the value of k?

(A) 1

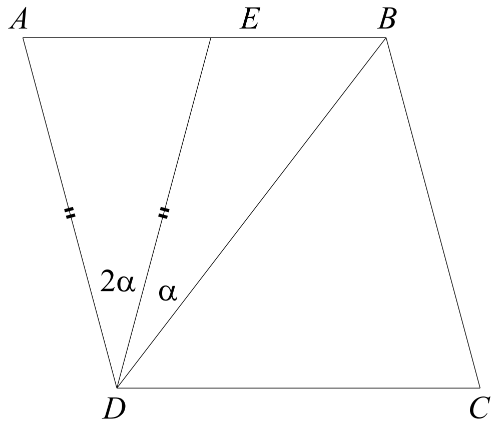
(B) -1

(C) 0

(D) 8

Exam continues next page ...

QUESTION TEN



The point E lies on the side AB of the rhombus $ABCD$ such that $AD = DE$. The angle ADE is 2α and the angle EDB is α. Find the value of α.

(A) 45°

(B) 30°

(C) 18°

(D) 15°

End of Section I

Exam continues overleaf ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION ELEVEN (15 marks) Use a separate writing booklet. **Marks**

(a) Integrate $\dfrac{3}{x}$ with respect to x. $\boxed{1}$

(b) Factorise $3x^2 - 7x + 2$. $\boxed{2}$

(c) Solve $\dfrac{5x-8}{x} = 1$. $\boxed{2}$

(d) Find the equation of the tangent to the curve $y = x^3 + 4$ at the point $(1, 5)$. $\boxed{2}$

(e) Differentiate $y = \cos(6x + 5)$. $\boxed{2}$

(f) Find the exact value of θ such that $\sin 2\theta = 1$, where $0 \le \theta \le \pi$. $\boxed{2}$

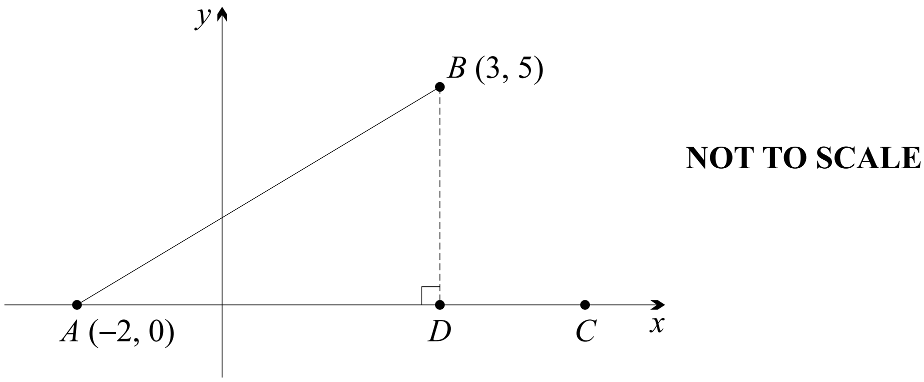
(g) A sector with radius 5 cm has an arc length of 20 cm. Find the area of the sector. $\boxed{2}$

(h) Find the limiting sum of the series $\dfrac{17}{3} + \dfrac{17}{9} + \dfrac{17}{27} + \cdots$. $\boxed{2}$

Exam continues next page ...

QUESTION TWELVE (15 marks) Use a separate writing booklet. **Marks**

(a)



The diagram shows the points $A(-2,0)$, $B(3,5)$ and the point C which lies on the x-axis. The point D also lies on the x-axis such that BD is perpendicular to AC.

(i) Show that the gradient of AB is 1. **1**

(ii) Find the equation of the line AB. **1**

(iii) What is the size of $\angle BAC$? **1**

(iv) The length of BC is 13 units. Find the length of DC. **1**

(v) Calculate the area of $\triangle ABC$. **1**

(vi) Calculate the size of $\angle ABC$, to the nearest degree. **2**

(b) A particle moves on a horizontal line so that its displacement x cm to the right of the origin at time t seconds is given by the function

$$x = \tfrac{1}{3}t^3 - 6t^2 + 27t - 18.$$

(i) Find the velocity function. **1**

(ii) When is the particle stationary? **1**

(iii) Find the acceleration function. **1**

(iv) When is the acceleration zero? **1**

(v) Where is the particle when the acceleration is zero? **1**

(c) A company starts with 60 employees. At the beginning of each subsequent year the number of employees increases by 15%.

(i) Find a formula for the number of employees at the beginning of the nth year. **1**

(ii) In which year did the number of employees first exceed 120? **2**

QUESTION THIRTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) (i) Find the values of m for which the equation $mx^2 - 4x + m = 0$ has real roots. **2**

(ii) (α) For what values of m does the equation $mx^2 - 4x + m = 0$ have one root only? **1**

(β) Find this root for each value of m in (α). **1**

(b) The rate of increase in the number of bacteria N in a culture after t hours is proportional to the number present. This can be represented by the differential equation $\dfrac{dN}{dt} = kN$. Initially there are 1000 bacteria present and two hours later there are 1080.

(i) Show that $N = 1000e^{kt}$, where k is a constant, is a solution to the differential equation $\dfrac{dN}{dt} = kN$. **1**

(ii) Find the exact value of k. **1**

(iii) Find the number of bacteria present after a further two hours. **2**

(iv) At what time will the culture have doubled its initial size? **2**

(c) Suppose that $f'(x) = \sin 2x$ and $f(\pi) = 1$.

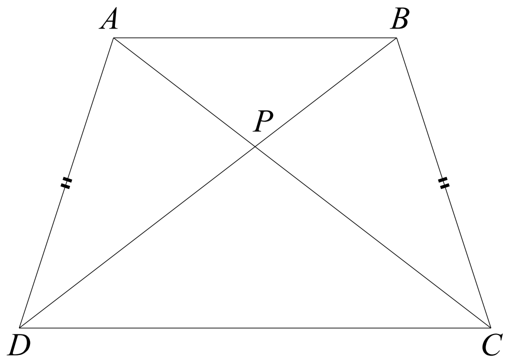
(i) Find the function $f(x)$. **3**

(ii) Find the exact value of $f(\frac{\pi}{3})$. **2**

Exam continues next page ...

QUESTION FOURTEEN (15 marks) Use a separate writing booklet. **Marks**

(a)



The quadrilateral $ABCD$ has diagonals AC and BD which intersect at P.
It is known that $AD = BC$ and $AC = BD$.
Copy the diagram into your answer booklet.

(i) Prove that the triangles ABC and BAD are congruent. **3**

(ii) Show that triangle ABP is isosceles. **2**

(iii) Hence show that triangle CDP is isosceles. **2**

(iv) Show that AB is parallel to CD. **3**

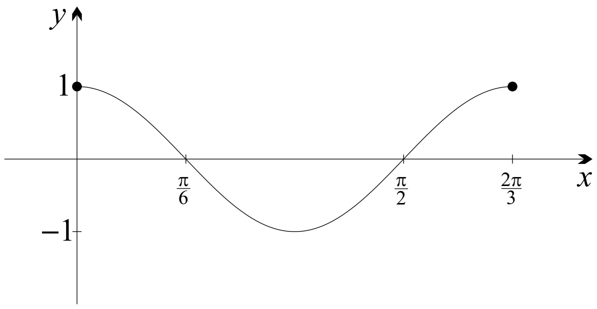
(b) (i) Find the gradient of the tangent to $y = \sin x$ at the origin. **1**

(ii) Draw the graphs of $y = \sin x$, $y = \frac{2}{3}x$ and the tangent in part (i). Draw your three graphs on the same set of axes for $0 \le x \le \pi$. **3**

(iii) For what values of m does the equation $\sin x = mx$ have a solution in the domain $0 < x < \pi$? **1**

QUESTION FIFTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) **3**



The diagram above shows the graph of the function $y = \cos 3x$. Find the total area bounded by $y = \cos 3x$ and the x-axis from $x = 0$ to $x = \frac{\pi}{3}$.

(b) If α and β are the roots of the quadratic equation $5x^2 - x - 3 = 0$, find the value of:

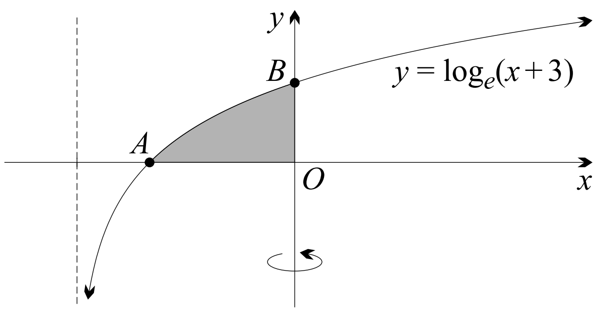
(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **1**

(iv) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **1**

(c)



The diagram shows the graph of the function $y = \log_e(x + 3)$. The graph crosses the axes at A and B as shown.

(i) Write down the coordinates of B. **1**

(ii) Write x as a function of y. **1**

(iii) Find the exact value of the volume generated when the shaded region AOB is rotated about the y-axis. **3**

QUESTION FIFTEEN (Continued)

(d) Consider the function given by $y = \sin^2 x$.

(i) Copy and complete the following table in your answer booklet. **1**

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y					

(ii) Use Simpson's rule with five function values to find an approximation to **2**

$$\int_0^{\pi} \sin^2 x \, dx.$$

The Exam continues on the next page

QUESTION SIXTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) A company borrows \$800 000 to update its car fleet. The interest rate is 12% p.a. compounded monthly. It pays off the loan by 24 equal monthly instalments. The first instalment is paid one month after the loan is taken out.
Let A_n be the amount owing after n instalments are paid. Let M be the amount of each instalment.

(i) Show that the amount owing after two months is $A_2 = 816\,080 - M(2{\cdot}01)$. **2**

(ii) Show that $M = \dfrac{8000 \times 1{\cdot}01^{24}}{1{\cdot}01^{24} - 1}$. **2**

(iii) Hence calculate M to the nearest dollar. **1**

(iv) After paying ten instalments, the company decides to increase its repayments to \$60 000 each month. Find the total number of months it takes the company to pay off its debt. **3**

(b) A van is to travel 1000 kilometres at a constant speed of v km/h.
When travelling at v km/h, the van uses fuel at a rate of $(6 + \frac{v^2}{50})$ litres per hour.
The truck company pays \$1.50 per litre for fuel and pays each of the two drivers \$30 per hour while the van is travelling.

(i) Let the total cost of fuel and the drivers' wages for the trip be C dollars. Show that **3**

$$C = \frac{69\,000}{v} + 30v.$$

(ii) The van must take no longer than 12 hours to complete the trip, and speed limits require that $v \leq 110$. **4**
At what speed v should the van travel to minimise the cost C?

End of Section II

END OF EXAMINATION

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The following list of standard integrals may be used:

$$\int x^n \, dx = \frac{1}{n+1} \, x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} \, e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

$$\text{NOTE : } \ln x = \log_e x, \; x > 0$$

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- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A ◯ B ◯ C ◯ D ◯

Question Two

A ◯ B ◯ C ◯ D ◯

Question Three

A ◯ B ◯ C ◯ D ◯

Question Four

A ◯ B ◯ C ◯ D ◯

Question Five

A ◯ B ◯ C ◯ D ◯

Question Six

A ◯ B ◯ C ◯ D ◯

Question Seven

A ◯ B ◯ C ◯ D ◯

Question Eight

A ◯ B ◯ C ◯ D ◯

Question Nine

A ◯ B ◯ C ◯ D ◯

Question Ten

A ◯ B ◯ C ◯ D ◯

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CANDIDATE NUMBER: ....

Question One

A ○ B ● C ○ D ○

Question Two

A ○ B ○ C ● D ○

Question Three

A ○ B ○ C ● D ○

Question Four

A ○ B ○ C ○ D ●

Question Five

A ○ B ○ C ○ D ●

Question Six

A ● B ○ C ○ D ○

Question Seven

A ○ B ● C ○ D ○

Question Eight

A ○ B ○ C ○ D ●

Question Nine

A ● B ○ C ○ D ○

Question Ten

A ○ B ○ C ● D ○

Solutions 2U Trial 2014

1. -2 B

2. $5{\cdot}30$ C

3. $(\sqrt{5}+2\sqrt{3})(\sqrt{5}-2\sqrt{3}) = 5-12 = -7$

$\frac{\sqrt{5}-2\sqrt{3}}{-7}$ or $\frac{2\sqrt{3}-\sqrt{5}}{7}$.

C

4. D

5. D

6. $y = e^{7x} \times e^{-3x}$

$= e^{4x}$

$y' = 4e^{4x}$

A

7. $x = 20t - 5t^2$

$\dot{x} = 20 - 10t = 0$

$t = 2$

B

8. AP $a = 31$, $d = 13$ $T_n = 226$.

$T_n = a + (n-1)d$

$226 = 31 + (n-1)13$

$= 31 + 13n - 13$

$226 = 18 + 13n$

$13n = 208$

$n = 16$.

D

9. $\int_0^4 (x+k)\,dx = 12.$

$\frac{x^2}{2} + kx \Big]_0^4 = 12$

$\frac{16}{2} + 4k = 12$

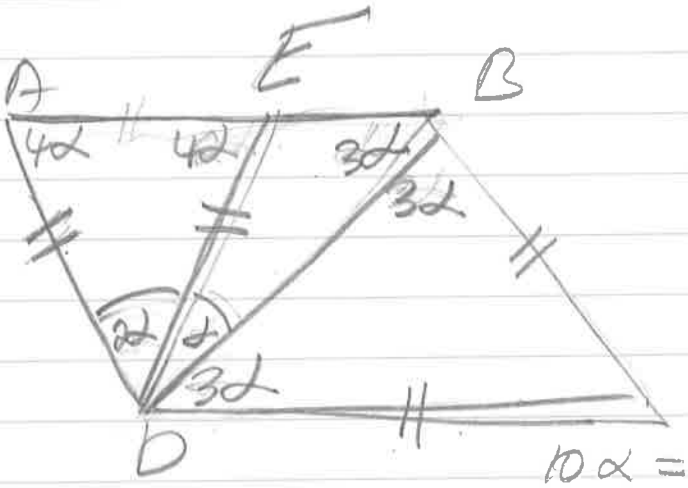
$4k = 12 - 8$

$= 4$

$k = 1$

A

10.



$10\alpha = 180°$

$\alpha = 18°.$

C.

11.

a) $\int \frac{3}{x}\,dx = 3\log_e x + c$ (c not required)

b) $3x^2 - 7x + 2 = (3x - 1)(x - 2)$

c) $5x - 8 = x$

$4x = 8$

$x = 2$

d) $y = x^3 + 4$

$\frac{dy}{dx} = 3x^2$

$x = 1, \quad m = 3$

$y - y_1 = m(x - x_1)$

$y - 5 = 3(x - 1)$

$y - 5 = 3x - 3$

$y = 3x + 2$

e) $y = \cos(6x + 5)$

$\frac{dy}{dx} = -6\sin(6x + 5)$

f) $\sin 2\theta = 1$

$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

(g) $l = r\theta$

$20 = 5\theta$

$A = \frac{1}{2} r^2 \theta, \quad (r\theta = 20, \; r = 5)$

$= \frac{1}{2} \times 5 \times 20$

$= 50 \text{ cm}^2$

(19 units)

h) $\frac{12}{3} + \frac{12}{9} + \frac{12}{27} + \ldots$

GP, $a = \frac{12}{3}$, $r = \frac{1}{3}$ ✓

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{\frac{12}{3}}{1-\frac{1}{3}}$$

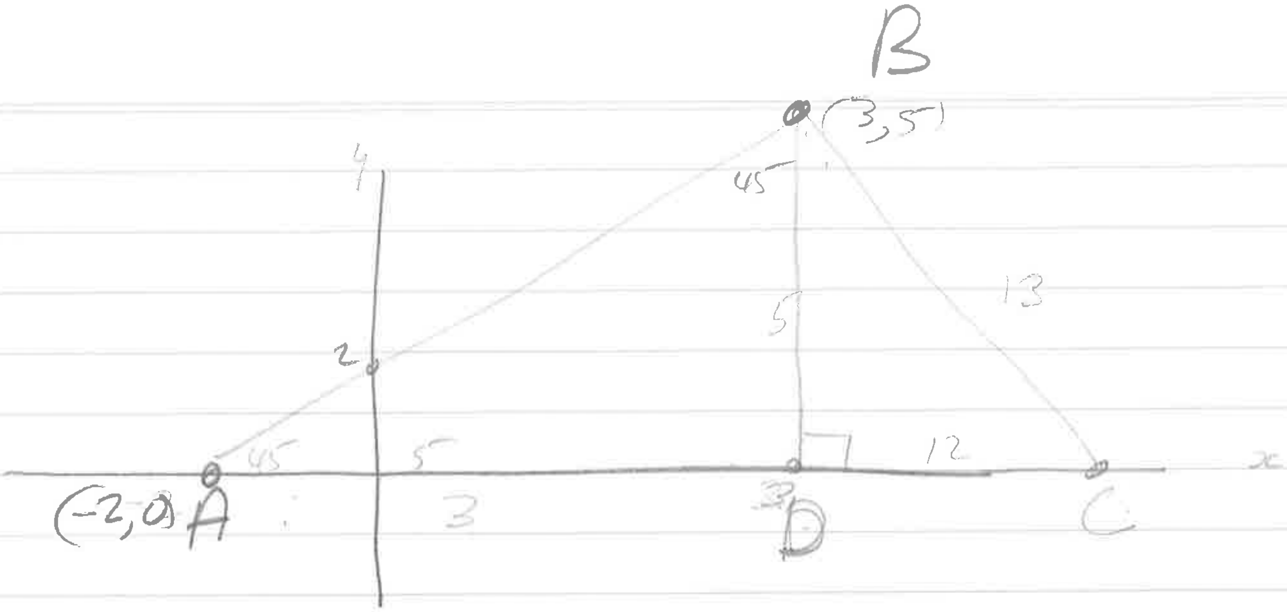
$$= \frac{\frac{12}{3}}{\frac{2}{3}}$$

$$= \frac{12}{3} \times \frac{3}{2}$$

$$= \frac{12}{2}.$$ ✓

12.

a)



(i) $\frac{5}{5}$ $m = \frac{y_1 - y_2}{x_1 - x_2}$

$= \frac{5-0}{3--2}$

$= 1$

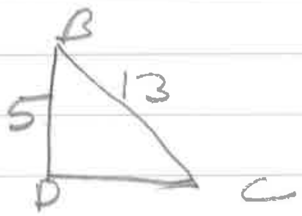
ii) $y - y_1 = m(x - x_1)$

$y - 0 = 1(x+2)$

$y = x + 2.$

iii) $\angle BAC = 45^0$

iv)



$DC^2 = 13^2 - 5^2$

$DC = 12$ u

v) $|AC| = 2 + 3 + 12 = 17$

area $= \frac{1}{2} \times 17 \times 5$

$= 42.5$ u^2.

(vi)

vi) $\sin \angle DBC = \frac{12}{13}$

$\angle DBC = 67^\circ 23'$

$\angle ABD = 45^\circ$

so $\angle ABC = 45^\circ + 67^\circ 23'$
$= 112^\circ 23'$
$\simeq 112^\circ$

OR use the sine rule.

b) $x = \frac{1}{3}t^3 - 6t^2 + 27t - 18$

(i) $\dot{x} = t^2 - 12t + 27$

(ii) $t^2 - 12t + 27 = 0$
$(t - 3)(t - 9) = 0$
$t = 3$ and 9 sec

(iii) $\ddot{x} = 2t - 12$

(iv) $t = 6$ seconds

(v) $x = \frac{1}{3} \times 6^3 - 6 \times 6^2 + 27 \times 6 - 18$
$= 0$ cm.
it is at the origin

c) (i) $N_2 = 60(1.15)$
$N_n = 60(1.15)^{n-1}$

(ii) $60(1.15)^{n-1} = 120$
$(1.15)^{n-1} = 2$

$60 \quad 60(1.15) \quad 60(1.15)^2 \quad 60(1.15)^3 \quad 60(1.15)^4 \quad 60(1.15)^5$

120.68.

$$(n-1)\log 1.15 = \log 2$$

$$n\log 1.15 - \log 1.15 = \log 2$$

$$n\log 1.15 = \log 2 + \log 1.15$$

$$n = \frac{\log 2 + \log 1.15}{\log 1.15}$$

$$= 5.959\ldots$$

OR

Guess/Check

During first year	60
" 2nd yr	$60(1.15)$
3rd yr	$60(1.15)^2$
4th yr	$60(1.15)^3$
5th yr	$60(1.15)^4 = 104.94$
6th yr	$60(1.15)^5 = 120.68.$

The number of employees exceeds 120 in the 6th year.

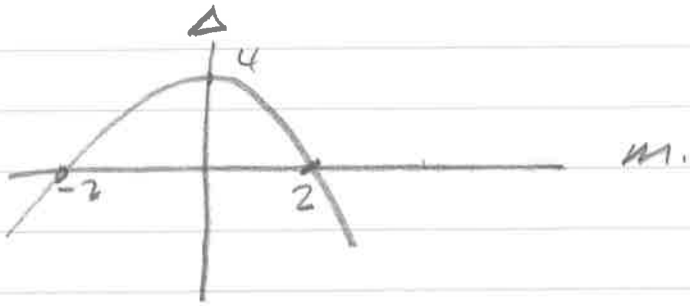
Q13.

a). (i) $mx^2 - 4x + m = 0$

for real roots $\Delta \geq 0$.

$\Delta = 16 - 4m^2 \geq 0$ ✓

$4(4 - m^2) \geq 0$.



$\Delta \geq 0$ for $-2 \leq m \leq 2$. ✓

(ii) (α) $mx^2 - 4x + m = 0$.

for one root, $\Delta = 0$.

also include soln in m & solve eqn.

$16 - 4m^2 = 0$

$m = 2$ or -2. ✓

(β) $x = \frac{-b}{2a}$

$= \frac{4}{2m}$

$m = 2$, $x = \frac{4}{4} = 1$

$m = -2$ $x = \frac{4}{-4} = -1$

✓ need both

b) $\frac{dN}{dt} = kN$

$t=0,\ N_0 = 1000$

$t=2,\ N_2 = 1080.$

(i) $N = 1000e^{kt}$

$\frac{dN}{dt} = k\,1000e^{kt}$

$= kN$

(ii) $N = 1000e^{kt}$

$t=2,\quad 1080 = 1000e^{2k}.$

$e^{2k} = 1.08$

$2k = \ln 1.08$

$k = \frac{1}{2}\ln 1.08$

or $e^{2k} = \frac{27}{25}.$

$2k = \frac{27}{25}$

$k = \frac{1}{2}\ln\frac{27}{25}$

(iii) $t=4,\quad N = 1000e^{2\ln 1.08}$

$= 1000 \times 1.08^2$

$= 1166.4.$

$\simeq 1166$

(iv) find t when $N = 2000$

$2000 = 1000e^{kt}.$

$2 = e^{kt}$

$kt = \ln 2$

$$t = \ln 2 \div \tfrac{1}{2}\ln 1.05$$

$\simeq$ 18 hours after initial time.

(c) $f'(x) = \sin 2x, \quad f(\pi) = 1.$

(i) $f(x) = \int \sin 2x \, dx$

$= -\tfrac{1}{2}\cos 2x + c$

$x = \pi,$ $\quad 1 = -\tfrac{1}{2}\cos 2\pi + c$

$1 = -\tfrac{1}{2} + c \quad$ so $\quad c = \tfrac{3}{2}$

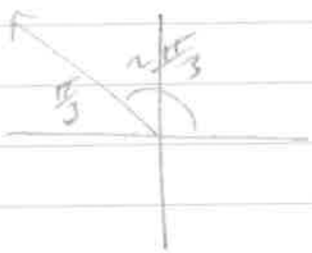
$f(x) = -\tfrac{1}{2}\cos 2x + \tfrac{3}{2}$

(ii) $f(\tfrac{\pi}{3}) = -\tfrac{1}{2}\cos\tfrac{2\pi}{3} + \tfrac{3}{2}$

$= -\tfrac{1}{2}\times(-\tfrac{1}{2}) + \tfrac{3}{2}$

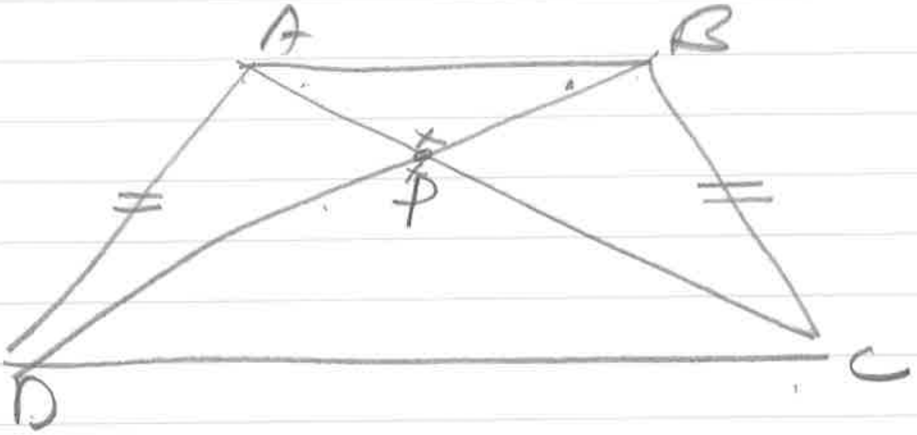
$= \tfrac{1}{4} + \tfrac{3}{2}$

$= \tfrac{7}{4}$



Q14.

a)



(i) In $\triangle ABC$, $\triangle BAD$.

AB is common

BC = AD, given

AC = BD given

$\therefore$ $\triangle ABC \equiv \triangle ABD$, SSS.

need reason

(ii) Matching angles in congruent triangles are equal

$\therefore$ $\angle ABD = \angle DAB$

so $\triangle APB$ is isosceles since it has two equal angles.

iii) Now AC = DB given

AP = BP, sides opposite equal angles in $\triangle APB$.

So AC − AP = BD − BP.

ie PD = PC and $\triangle DPC$ is isosceles.

(can also show $\angle D = \angle C$).

iv) $\angle APB = \angle DPC$, vertically opposite

So $\angle PAB + \angle ABP = \angle PDC + \angle PCD$, angle sum of $\triangle APB$, $\triangle DPC$ are 180°

But both are isosceles

So $\angle ABP = \angle PDC$

But these are alternate

So $AB \parallel DC$

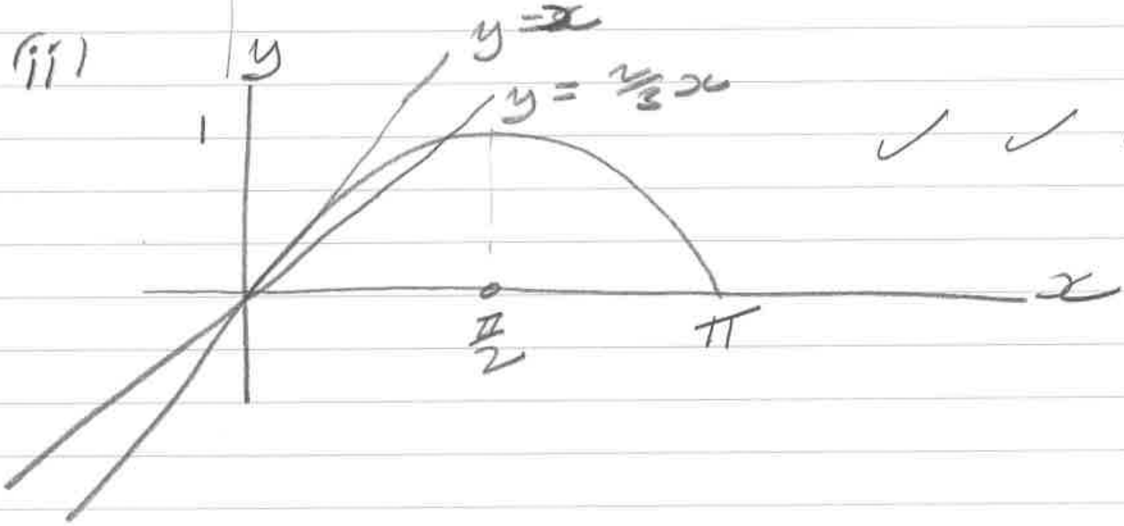
(b) (i) $y = \sin x$

$\frac{dy}{dx} = \cos x$

$x = 0$, $m = \cos 0$

so $m = 1$ (can just state $m = 1$).

(ii)



(iii) $0 < m < 1$.

Q15

a) $$A = 2\int_0^{\frac{\pi}{6}} \cos 3x \, dx$$

$$= \frac{2}{3}\left[\sin 3x\right]_0^{\frac{\pi}{6}}$$

$$= \frac{2}{3}\left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$= \frac{2}{3}\, u^2$$

b) $$5x^2 - x + 3 = 0$$

(i) $$\alpha + \beta = \frac{1}{5}$$

(ii) $$\alpha\beta = \frac{-3}{5}$$

(iii) $$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

$$= \frac{1}{25} + \frac{6}{5}$$

$$= \frac{31}{25}$$

(v) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{\frac{1}{5}}{-\frac{3}{5}}$

$= \frac{1}{5} \times -\frac{5}{3}$

$= -\frac{1}{3}$.

(c) (i) B is $(0, \ln 3)$.

(ii) $y = \ln(x+3)$.

$e^y = x+3$.

$x = e^y - 3$.

(iii) $V = \pi \int_0^{\ln 3} x^2 \, dy$

$= \pi \int_0^{\ln 3} (e^y - 3)^2 \, dy$

$= \pi \int_0^{\ln 3} e^{2y} - 6e^y + 9 \, dy$

$= \pi \left[\frac{1}{2}e^{2y} - 6e^y + 9y \right]_0^{\ln 3}$

$= \pi \left[\left(\frac{1}{2} \times 9 - 6 \times 3 + 9\ln 3\right) - \left(\frac{1}{2} - 6 + 0\right) \right]$

$= \pi \left(4\frac{1}{2} - 18 + 9\ln 3 + 5\frac{1}{2} \right)$

$= \pi (-8 + 9\ln 3)$.

(d) $y = \sin^2 x$

$\sin \frac{3\pi}{4}$

$\frac{\pi}{2}$ $\frac{\pi}{2}$

(i)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	π
y	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0.

(ii) $\int_0^{\pi} \sin^2 x \, dx \doteq \frac{\frac{\pi}{2}}{6}\left(0 + \frac{1}{2} + 1 + 1 + \frac{1}{2} + 0\right)$

$= \frac{\pi}{12} \times 6$

$= \frac{\pi}{2}$ u^2

Q16

(a)

i) $A_1 = 800\,000(1.01) - M$

$A_2 = 800\,000(1.01)^2 - M(1.01) - M$

$= 800\,000(1.01)^2 - M(1.01 + 1)$

$= 800\,000(1.01)^2 - M(2.01)$

(ii)

$A_{24} = 800\,000(1.01)^{24} - M(1.01^{23} + 1.01^{22} + \cdots + 1)$

when loan is paid off $A = 0$

so $800000(1.01)^{24} - M(1.01^{23} + 1.01^{22} + \cdots + 1) = 0$

• need to show GP for this mk.

$$800\,000(1.01)^{24} = \frac{M(1.01^{24} - 1)}{0.01}$$

• need to see $A_{24} = 0$ or some variation of it.

$$M = \frac{800\,000(1.01)^{24} \times 0.01}{(1.01)^{24} - 1}$$

$$= \frac{8000(1.01)^{24}}{(1.01)^{24} - 1}.$$

iii) $M \simeq \$37\,659$

iv). $A_{10} = 800\,000(1.01)^{10} - 37659\left(\frac{1.01^{10}-1}{0.01}\right)$.

$= \$489701$

So we need to pay this off with
$M = \$60\,000$

$$0 = 489701(1.01)^n - 60\,000\left(\frac{1.01^n - 1}{0.01}\right).$$

$$489701(1.01)^n = 6000000(1.01^n) - 6000000.$$

$$(1.01)^n(6000000 - 489701) = 6000000$$

$$(1.01)^n = \frac{6000000}{5510299}$$

$$= 1.08887.$$

$$n = \frac{\ln(1.08887)}{\ln 1.01}$$

$$= 8.5$$

$$\simeq 9 \quad \text{months!}$$

It takes 19 months to pay off the debt.

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(b) (i) Time for trip $= \frac{1000}{v}$ hrs

Drivers' wages $= \frac{30 \times 1000}{v} \times 2$

No of litres of fuel $= \$1.50 \times \frac{1000}{v} \times \left(6 \times \frac{v^2}{50}\right)$

$= \frac{1500}{v} \times \left(6 \times \frac{v^2}{50}\right)$

$= \frac{9000}{v} + 30v$

So Total $= \frac{60000}{v} + \frac{9000}{v} + 30v$

$C = \frac{69000}{v} + 30v$ dollars.

(ii) $C = 69000v^{-1} + 30v$

$\frac{dC}{dv} = -69000v^{-2} + 30$

$= 0$ at stationary point

$\frac{69000}{v^2} = 30$

$v^2 = 2300$

$v = \sqrt{2300}$

$\simeq 48$ km/hr.

Now if $v = \sqrt{2300}$, then $t = \frac{1000}{\sqrt{2300}}$ hrs

$\simeq 20.8$ hours.

This is too long, we must take no longer than 12 hours.

We want $\frac{1000}{v} \leq 12$

$$v \geq \frac{1000}{12}$$

$$v \geq 83\tfrac{1}{3} \text{ km/hr}$$

If $v = 83\frac{1}{2}$, $C = \frac{69000}{83\frac{1}{2}} + 30 \times 83\frac{1}{3}$

$$= \$3328$$

$v = 110$, $C = \frac{69000}{110} + 30 \times 110$

$$= \$3927$$

check concavity: $\frac{d^2C}{dv^2} = 2 \times 69000\, v^{-3} > 0$

for all $v > 0$

So $v = 48$ km/hr gives a minimum turning point, and C is increasing after $v = 48$. As v increases C increases, so, the minimum C that satisfies the conditions is $v = 83\frac{1}{3}$ km/hr.